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For consideration at the Radio Section

On the transmission capacity of 'ether' and wire in electric communications *

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Both in radio and in wire communication technology, every transmission requires a certain frequency range rather than some single frequency. This has the effect that only a limited number of radio stations (broadcasting different programs) can operate simultaneously. Along one pair of wires it is also impossible to transfer at once more than a certain number of transmissions, because the frequency band of one transmission should not overlap with the band of another one, for such an overlap would lead to mutual interference.

To increase the transmission capacity of 'ether' and wire (this would be of enormous practical significance, especially in view of the rapid development of radio engineering and such broadcasts as television) necessitates narrowing somehow the frequency range required for a given broadcast, without degrading its quality, or inventing a way of separating broadcasts not on the basis of frequency, as has been done up till now, but on some other basis. [1]

Presently, no contrivances in these realms have made it possible to increase, even theoretically, the transmission

* The paper of 1933 is reproduced from the edition published on the 70th anniversary of the Kotel'nikov theorem and the 95th birthday of Vladimir Aleksandrovich Kotel'nikov by the Institute of Radioengineering and Electronics of the Moscow Power Engineering Institute (Technical University) in 2003 under the supervision of its Director N N Udalov. Minor alterations have been made in the reproduction: formulas have been written in the format adopted by *Phys.-Usp.*, the page numbering of footnotes replaced with a continuous one, the orthography and the syntax brought into agreement with modern standards. The author's style has been retained.

[1] True, this can sometimes be effected with directional antennas, but here we will consider only the case when this cannot be done with antennas for some reason or other.

capacity of 'ether' and wire to a greater degree than is allowed by transmitting 'on one sideband'.

This brings up the question of whether this can be done at all. Or is it that all attempts in this area will be equivalent to attempts to construct a 'perpetuum mobile'?

This radio engineering problem is topical nowadays in view of the 'tightness in the ether', which is growing year after year. It is important to now elucidate this question in connection with the planning of scientific research, because in the planning it is vital to know what is possible and what is absolutely impossible to do, so as to mount efforts in the right direction.

In the present work I tackle this problem and prove that there exists a quite specific, minimally necessary frequency band for television and the transmission of images with all their half-shadows, as well as for telephone communication. This frequency band can in no way be narrowed without degrading the rate and quality of transmission. It is also proven that for these broadcasts there is no way of increasing the transmission capacity of either 'ether' or wire by applying nonfrequency selection of any kind or any other means (with the exception, of course, of the selection by directions employing directional antennas). The maximum achievable transmission capacity for these broadcasts may be obtained in the transmission 'on one sideband', and basically this is quite attainable at present.

For such transmissions as telegraphy or the image transmission and television without penumbra, etc., in which the transmitted object may assume only definite values known in advance and not vary continuously, it is shown that the requisite frequency band can be reduced by an arbitrarily large factor without impairing the quality of transmission or its rate, this being achieved by increasing the power and complexity of the equipment. One method for such a frequency band reduction is pointed out in this paper, and it is shown what increase in power is required for this purpose.

Therefore, no theoretical limit is imposed on the transmission capacity of 'ether' and wire for broadcasts of this kind; the problem is only one of technical implementation.

In the present work, the above propositions are proven without reference to a broadcast technique on the following basis: in all kinds of electric communication, the transmitter can send and the receiver can bring in only signals being some function of time which cannot be absolutely arbitrary, because the frequencies it consists of and may be resolved into must be confined to certain ranges. In a radio broadcasting, this function is the current intensity of the transmitting antenna, which is perceived by the receiver more or less accurately; in a wire communication, this is the electromotive force at the origin of the line. In either case, the transmitted functions would comprise frequencies from a limited range because, first, very high and very low frequencies would not reach the receiver due to propagation conditions and, second, ordinarily the frequencies that are beyond a prescribed narrow range are purposely suppressed, so as not to be a hindrance to other broadcasts.

The inevitability of transmission with the help of time functions that contain only a limited frequency range entails, as shown below, quite a definite restriction on the transmission capacity.

To prove the stated propositions, we address ourselves to the study of functions consisting of a definite frequency range.

Functions consisting of frequencies from 0 to f_1

Theorem I. Any function $F(t)$ consisting of frequencies from 0 to f_1 cycles per second can be represented as a series

$$F(t) = \sum_{-\infty}^{+\infty} D_k \frac{\sin \omega_1 [t - k/(2f_1)]}{t - k/(2f_1)}, \tag{1}$$

where k is an integer, $\omega_1 = 2\pi f_1$, and D_k are constants depending on $F(t)$.

And vice versa, any function $F(t)$ represented as a series (1) consists of only the frequencies from 0 to f_1 cycles per second.

Proof. Any function $F(t)$ subject to the Dirichlet conditions (a finite number of maxima, minima, and discontinuity points on any finite segment) and integrable between the limits from $-\infty$ to $+\infty$, which is always the case in electrical engineering, can be represented as a Fourier integral [2,3]:

$$F(t) = \int_0^{\infty} C(\omega) \cos \omega t \, \mathrm{d}\omega + \int_0^{\infty} S(\omega) \sin \omega t \, \mathrm{d}\omega, \tag{2}$$

i.e., as the sum of an infinite number of sinusoidal oscillations with frequencies from 0 to ∞ and the frequency-dependent amplitudes $C(\omega)$ and $S(\omega)$. In this case, one obtains

$$C(\omega) = \frac{1}{\pi} \int_{-\infty}^{+\infty} F(t) \cos \omega t \, \mathrm{d}t, \qquad S(\omega) = \frac{1}{\pi} \int_{-\infty}^{+\infty} F(t) \sin \omega t \, \mathrm{d}t. \tag{3}$$

In our case, when $F(t)$ consists of only the frequencies from 0 to f_1, evidently one finds

$$C(\omega) = 0, \qquad S(\omega) = 0$$

for

$$\omega > \omega_1 = 2\pi f_1,$$

and $F(t)$ can therefore be represented according to Eqn (2) as follows:

$$F(t) = \int_0^{\omega_1} C(\omega) \cos \omega t \, \mathrm{d}\omega + \int_0^{\omega_1} S(\omega) \sin \omega t \, \mathrm{d}\omega. \tag{4}$$

The functions $C(\omega)$ and $S(\omega)$, like any other ones, may always be represented as Fourier series on the interval

$$0 < \omega < \omega_1.$$

In this case, these series may, at our will, consist of only cosines or only sines, provided the double length of the interval is taken as the period, i.e., $2\omega_1$ [4]. Therefore, one has

$$C(\omega) = \sum_0^{\infty} A_k \cos \frac{2\pi}{2\omega_1} k\omega \tag{5a}$$

[2] See, for instance, V I Smirnov, *Course of Higher Mathematics Vol. II*, 1931 publ., p. 427.

[3] In what follows we also consider only the functions satisfying the Dirichlet conditions.

[4] See, for instance, V I Smirnov, *Course of Higher Mathematics Vol. II*, 1931 publ., p. 385.

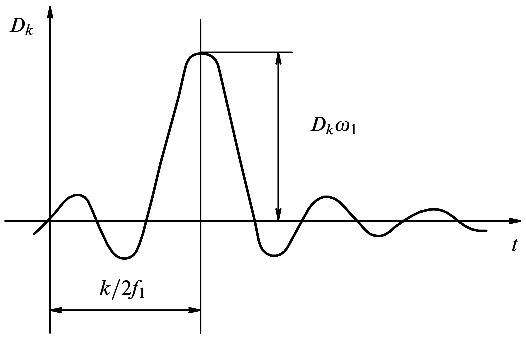

Figure 1.

and

$$S(\omega) = \sum_0^{\infty} B_k \sin \frac{2\pi}{2\omega_1} k\omega \,. \tag{5b}$$

We introduce the following notation

$$D_k = \frac{A_k + B_k}{2}\,, \qquad D_{-k} = \frac{A_k - B_k}{2}\,; \tag{6}$$

formulas (5a) and (5b) can then be rewritten as

$$C(\omega) = \sum_{-\infty}^{+\infty} D_k \cos \frac{\pi}{\omega_1} k\omega\,, \qquad S(\omega) = \sum_{-\infty}^{+\infty} D_k \sin \frac{\pi}{\omega_1} k\omega\,. \tag{7}$$

On substituting expressions (7) into formula (4), on some rearrangements and integration (see Appendix I) we obtain Eqn (1), i.e., prove the first part of Theorem I.

To prove the second part of the theorem, we consider the special case of $F(t)$ when the spectrum of its constituent frequencies falls in the range 0 to f_1 and is expressed in the form of Eqn (7) in which all D_k, with the exception of one, are equal to zero. This $F(t)$ will evidently consist of a single term of series (1). And vice versa: when $F(t)$ consists of a single, arbitrary term of series (1), its entire spectrum is confined to the range 0 to f_1. And therefore the sum of any individual terms of series (1), i.e., series (1) itself, will consist of frequencies confined to the range 0 to f_1, which proves the statement.

All terms of series (1) are similar and differ by only the shift in time and the factors D_k. One of the terms with a subscript k is plotted in Fig. 1; it peaks for $t = k/(2f_1)$ and possesses a gradually decreasing amplitude in both directions.

Theorem II. Any function $F(t)$ consisting of frequencies from 0 to f_1 can be continuously transmitted with an arbitrary accuracy with the aid of numbers which follow one after another $1/(2f_1)$ seconds apart. Indeed, by measuring the value of $F(t)$ for $t = n/(2f_1)$ (n is an integer), we will obtain

$$F\left(\frac{n}{2f_1}\right) = D_n\omega_1 \,. \tag{8}$$

Since all terms of series (1) vanish for this value of t, with the exception of the term with $k = n$, which, as is easily obtained by removing ambiguity, is equal to $D_n\,\omega_1$, in every $1/(2f_1)$th second we will be able to learn the next D_k. By transmitting these D_k one after another at $1/(2f_1)$-second intervals, from them we will be able, according to Eqn (1), to reconstruct $F(t)$ with an arbitrary accuracy.

Theorem III. It is possible to continuously and uniformly transmit arbitrary numbers D_k with a rate of N numbers per second by means of a function $F(t)$ with arbitrarily small items at frequencies greater than $f_1 = N/2$.

Indeed, on receiving every number we will construct the function $F_k(t)$ such that

$$\begin{aligned}
&\text{for}\quad t < \frac{k}{2f_1} - T \qquad F_k(t) = 0\,,\\
&\text{for}\quad \frac{k}{2f_1} - T < t < \frac{k}{2f_1} + T\\
&\qquad F_k(t) = D_k\,\frac{\sin\omega_1\big(t - k/(2f_1)\big)}{t - k/(2f_1)}\,,\\
&\text{for}\quad t > \frac{k}{2f_1} + T \qquad F_k(t) = 0\,,
\end{aligned} \tag{9}$$

and transmit their sum $F(t)$. Should be $T = \infty$, the resultant function $F(t)$ would consist only of frequencies lower than f_1, because in this case we would have obtained the series (1), but unfortunately such infinite series of terms are impossible to construct, and we will therefore restrict ourselves to finite T. We will prove the following: the longer T, the lower are the amplitudes at frequencies $f > f_1$, and these amplitudes can be made as small as desired. To this end, we will find the amplitudes $C(\omega)$ and $S(\omega)$ for function (9) by substituting it into Eqn (3). We obtain

$$\begin{aligned}
C(\omega) &= \frac{1}{\pi}\int_{k/(2f_1)-T}^{k/(2f_1)+T} D_k\,\frac{\sin\omega_1\big(t - k/(2f_1)\big)}{t - k/(2f_1)}\cos\omega t\,\mathrm{d}t\,,\\
S(\omega) &= \frac{1}{\pi}\int_{k/(2f_1)-T}^{k/(2f_1)+T} D_k\,\frac{\sin\omega_1\big(t - k/(2f_1)\big)}{t - k/(2f_1)}\sin\omega t\,\mathrm{d}t\,.
\end{aligned} \tag{10}$$

Upon integration (see Appendix II) we will have

$$\begin{aligned}
C(\omega) &= \frac{D_k}{\pi}\cos\omega\,\frac{k}{2f_1}\big[\mathrm{Si}\,T(\omega+\omega_1) - \mathrm{Si}\,T(\omega-\omega_1)\big]\,,\\
S(\omega) &= \frac{D_k}{\pi}\sin\omega\,\frac{k}{2f_1}\big[\mathrm{Si}\,T(\omega+\omega_1) - \mathrm{Si}\,T(\omega-\omega_1)\big]\,.
\end{aligned} \tag{11}$$

In this expression, Si denotes integral sine, i.e., the function

$$\mathrm{Si}\,x = \int_0^x \frac{\sin y}{y}\,\mathrm{d}y\,. \tag{12}$$

The values of this function were calculated and tabulated [5], and it is graphically displayed in Fig. 2.

As may be seen from Fig. 2, $\mathrm{Si}\,x$ tends to $\pm\pi/2$ as $x \to \pm\infty$.

We now consider the value of the expression in square brackets in expression (11). Its graphic representation for $T = 3/(2f_1)$ is given in Fig. 3a, for $T = 6/(2f_1)$ in Fig. 3b, for $T = 24/(2f_1)$ in Fig. 3c, and for $T = \infty$ in Fig. 3d.

As is evident from these plots, with increasing T the expression in square brackets in expression (11) tends to the

[5] See, for instance, E Jahnke und F Emde, *Funktionentafeln mit Formeln und Kurven.*

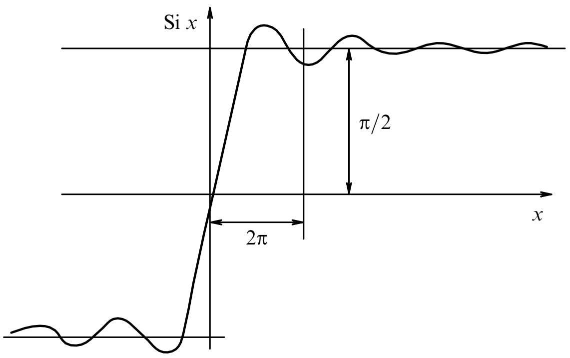

Figure 2.

limits in Fig. 3d, namely

for $\quad \omega > \omega_1 \quad [\ \ ] = 0\,,$

for $\quad \omega < \omega_1 \quad [\ \ ] = \pi\,.$

This is also clear directly from expression (11): with increasing T, it is as if the scale of ω increases and Si becomes a rapidly decaying function.

Therefore, the resultant sum of $F_k(t)$ will possess arbitrarily low amplitudes at frequencies $f > f_1$ provided T is taken sufficiently long.

On receiving the $F(t)$ function it is easy to recover the D_k numbers transmitted: at $t = n/(2f_1)$ all terms vanish with the exception of the term for which $k = n$, which is equal to $D_n\,\omega$.

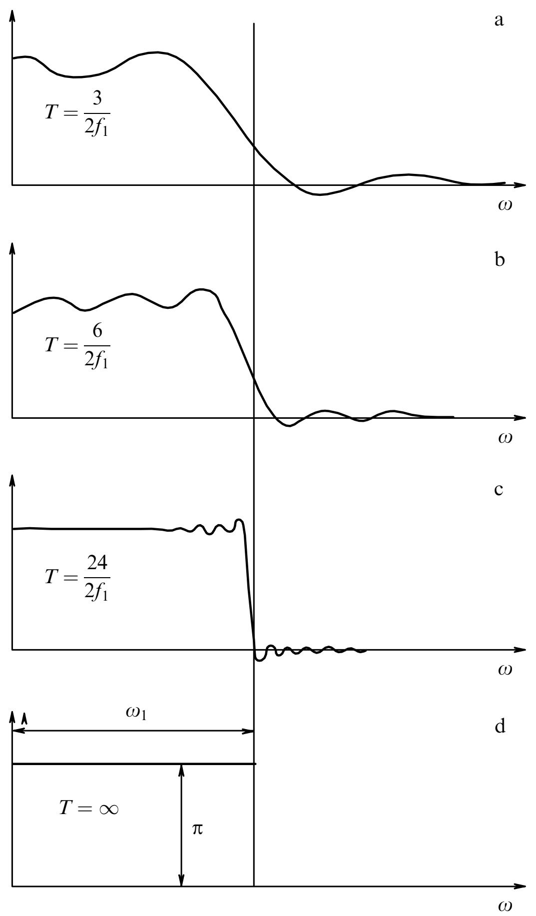

Figure 3.

And so

$$F\left(\frac{n}{2f_1}\right) = D_n\,\omega\,.$$

Therefore, from our function we will be able, by measuring its value at $t = k/(2f_1)$, to recover every $t = 1/(2f_1)$th second the value of a new D_k and to obtain $N = 2f_1$ transmitted numbers per second, which proves the statement.

Functions consisting of frequencies from f_1 to f_2
Let us prove a theorem.

Theorem IV. Any function $F(t)$ consisting of frequencies from f_1 to f_2 may be represented as

$$F(t) = F_1(t)\cos\frac{\omega_2+\omega_1}{2}\,t + F_2(t)\sin\frac{\omega_2+\omega_1}{2}\,t\,, \qquad (13)$$

where $\omega_1 = 2\pi f_1$, $\omega_2 = 2\pi f_2$, while $F_1(t)$ and $F_2(t)$ are some functions consisting of frequencies from 0 to $f = (f_2 - f_1)/2$. And vice versa: if $F_1(t)$ and $F_2(t)$ in Eqn (13) are arbitrary functions consisting of frequencies from 0 to $f = (f_2 - f_1)/2$, then $F(t)$ consists of frequencies from f_1 to f_2.

If $F(t)$ consists only of frequencies from f_1 to f_2, clearly $C(\omega)$ and $S(\omega)$ for this function may then be represented as follows:

$$C(\omega) = S(\omega) = 0 \quad \text{for} \quad \omega > \omega_2 \quad \text{or} \quad \omega < \omega_1\,,$$

$$\left.\begin{aligned} C(\omega) &= \sum_0^\infty A_k\cos\frac{\pi k}{2(\omega_2-\omega_1)}\,(\omega-\omega_1) \\ S(\omega) &= \sum_0^\infty B_k\sin\frac{\pi k}{2(\omega_2-\omega_1)}\,(\omega-\omega_1) \end{aligned}\right\} \text{for } \omega_1 < \omega < \omega_2,$$

or, introducing once again the notation

$$\begin{aligned} D_k &= \frac{A_k + B_k}{2}\,, \\ D_{-k} &= \frac{A_k - B_k}{2}\,, \end{aligned} \qquad (6)$$

we arrive at

$$\begin{aligned} C(\omega) &= \sum_{-\infty}^{+\infty} D_k\cos\frac{\pi}{\omega_2-\omega_1}\,k(\omega-\omega_1)\,, \\ S(\omega) &= \sum_{-\infty}^{+\infty} D_k\sin\frac{\pi}{\omega_2-\omega_1}\,k(\omega-\omega_1) \qquad (14) \\ &\text{for} \quad \omega_1 < \omega < \omega_2 \end{aligned}$$

and

$$C(\omega) = S(\omega) = 0 \quad \text{for} \quad \omega > \omega_2 \quad \text{or} \quad \omega < \omega_1\,. \qquad (14)$$

By substituting Eqn (14) into Eqn (2), upon integration and some rearrangement (see Appendix III) we obtain

$$\begin{aligned} F(t) = {} & \left[2\sum_{-\infty}^{+\infty}(-1)^n D_{2n}\,\frac{\sin(\omega_2-\omega_1)/2\{t-k/(f_2-f_1)\}}{t-k/(f_2-f_1)}\right] \\ & \times\cos\frac{\omega_2+\omega_1}{2}\,t \\ & + \left[2\sum_{-\infty}^{+\infty}(-1)^n D_{2n+1}\,\frac{\sin(\omega_2-\omega_1)/2\{t-(k+1/2)/(f_2-f_1)\}}{t-(k+1/2)/(f_2-f_1)}\right] \\ & \times\sin\frac{\omega_2+\omega_1}{2}\,t\,, \end{aligned} \qquad (15)$$

or, denoting

$$F_1(t) = 2\sum_{-\infty}^{+\infty}(-1)^n D_{2n}\,\frac{\sin(\omega_2-\omega_1)/2\big[t-k/(f_2-f_1)\big]}{t-k/(f_2-f_1)}\,, \tag{16}$$

$$F_2(t) = 2\sum_{-\infty}^{+\infty}(-1)^n D_{2n+1} \times\frac{\sin(\omega_2-\omega_1)/2\big[t-(k+1/2)/(f_2-f_1)\big]}{t-(k+1/2)/(f_2-f_1)} \tag{17}$$

and taking into account that the spectra of $F_1(t)$ and $F_2(t)$ must, according to Theorem I, necessarily consist of frequencies from 0 to $f=(f_2-f_1)/2$, because series (16) and (17) differ from series (1) in only the notation, the first part of Theorem IV may be considered proven.

Since any functions $F_1(t)$ and $F_2(t)$ consisting of frequencies from 0 to $f=(f_2-f_1)/2$ may, according to Theorem I, be represented by series (16) and (17) and since no constraints are imposed on the coefficients D_k that appear in these series, evidently the second part of Theorem IV is also valid.

We now prove two theorems which are a generalization of Theorems II and III.

Theorem V. Any function $F(t)$ which consists of frequencies from f_1 to f_2 may be continuously transmitted with an arbitrary accuracy by means of numbers transmitted one after another at $1/[2(f_2-f_1)]$-second intervals.

Indeed, for $t=k/(f_2+f_1)$ (k is an integer) we obtain according to formula (13):

$$F\left(\frac{k}{f_2+f_1}\right) = F_1\left(\frac{k}{f_2+f_1}\right), \tag{18}$$

because for this value of t the cosine is equal to unity, and the sine to zero. When $t=(k+1/2)/(f_2+f_1)$, by the same reasoning we obtain

$$F\left(\frac{k+1/2}{f_2+f_1}\right) = F_2\left(\frac{k+1/2}{f_2+f_1}\right).$$

Therefore, every $1/(f_2+f_1)$th second we will be able to learn the values of $F_1(t)$ and $F_2(t)$ one by one. From these values we will be able to reproduce the $F_1(t)$ and $F_2(t)$ functions themselves, because from so frequent a succession of numbers it is, according to Theorem II, possible to reproduce the functions consisting of frequencies from 0 to $(f_2+f_1)/2$, whereas the $F_1(t)$ and $F_2(t)$ functions consist only of frequencies from 0 to $(f_2-f_1)/2$.

Each of the functions thus obtained may, as a function consisting of frequencies from 0 to $(f_2-f_1)/2$, be transmitted, according to Theorem II, by numbers that follow one after another $1/(f_2-f_1)$ seconds apart; while evidently these two functions may be simultaneously transmitted by numbers that follow one after another $1/[2\,(f_2-f_1)]$ seconds apart. By first reconstructing $F_1(t)$ and $F_2(t)$ from these numbers, we will then be able to reconstruct $F(t)$ itself by formula (13).

Theorem VI. It is possible to continuously and uniformly transmit arbitrary numbers D_k with a rate N numbers per second by means of a function $F(t)$ that possesses arbitrarily small terms at frequencies $f>f_2$ and $f<f_1$ (i.e., is practically devoid of them) if

$$N = 2(f_2-f_1)\,. \tag{19}$$

Indeed, according to Theorem III we may transmit N numbers per second using two functions $F_1(t)$ and $F_2(t)$, each having arbitrarily small terms at frequencies above $(f_2-f_1)/2$.

The same functions may be continuously transmitted by the function $F(t)$ with arbitrarily small terms at frequencies $f>f_2$ and $f<f_1$. Indeed, from the functions $F_1(t)$ and $F_2(t)$ we will, according to Eqn (13), obtain $F(t)$, by the transmission of which we will be able, as noted above, to reconstruct $F_1(t)$ and $F_2(t)$ from it and thereby the numbers being transmitted.

To prove the last theorem, which states that there is no way to transmit infinitely much with the aid of a function comprising a limited frequency range, we will prove the following lemma.

Lemma. There is no way to transmit N arbitrary numbers with the aid of M numbers if

$$M < N\,. \tag{20}$$

Let us assume that this is possible to do.

Then, it is apparent that M numbers $m_1,\ldots,m_M$ are some functions of N numbers $n_1,\ldots,n_N$, namely

$$\begin{aligned} m_1 &= \varphi_1(n_1,\ldots,n_N)\,,\\ m_2 &= \varphi_2(n_1,\ldots,n_N)\,,\\ &\cdots\cdots\cdots\cdots\\ m_M &= \varphi_M(n_1,\ldots,n_N)\,, \end{aligned} \tag{21}$$

and we evidently should, with only the knowledge of the M numbers $m_1,\ldots,m_M$, and, of course, of the functions $\varphi_1,\ldots,\varphi_M$, manage to recover the numbers $n_1,\ldots,n_N$ from them.

But this is equivalent to the solution of M equations (21) with N unknown quantities, which is impossible to do when the number of equations is smaller than the number of unknowns, i.e., when inequality (20) holds.

Theorem VII. It is possible to continuously transmit arbitrary numbers which uniformly follow one after another with a rate of N numbers per second and M arbitrary functions $F_1(t),\ldots,F_M(t)$ with frequency ranges of widths $\Delta f_1,\ldots,\Delta f_M$ by means of numbers which continuously follow one after another with a rate of N' numbers per second and by means of M' functions $F_1'(t),\ldots,F_{M'}'(t)$ with the frequency ranges $\Delta f_1',\ldots,\Delta f_{M'}'$ if

$$N + 2\sum_1^M \Delta f_k \leqslant N' + 2\sum_1^{M'}\Delta f_k'\,. \tag{22}$$

And this cannot be done by any means when

$$N + 2\sum_1^M \Delta f_k > N' + 2\sum_1^{M'}\Delta f_k'\,. \tag{23}$$

The first part of this theorem is proved on the basis of Theorems V and VI.

Indeed, by virtue of Theorem V we can transmit our N numbers per second and M curves by means of P numbers per second when

$$P = N + 2\sum_{1}^{M} \Delta f_k\,. \tag{24}$$

And these P numbers per second may be partly transmitted by means of N' numbers per second and partly by means of the curves $F_1'(t), \ldots, F_{M'}'(t)$ on the strength of Theorem VI if equality (22) is correct.

The second part of the theorem will be proved by contradiction, on the basis of the lemma.

Assume that it is required to transmit P arbitrary numbers per second; according to Theorem VI this can be done by transmitting N numbers per second and the functions $F_1(t), \ldots, F_M(t)$ with the frequency ranges $\Delta f_1, \ldots, \Delta f_M$ if equality (24) is valid.

Were the second part of the theorem not valid, these functions and numbers would be possible to transmit by means of functions $F_1'(t), \ldots, F_{M'}'(t)$ and N' numbers per second. But the latter numbers and functions may, according to Theorem V, be transmitted by means of P' numbers per second if

$$P' = N' + 2\sum_{1}^{M'} \Delta f_k'\,. \tag{25}$$

In other words, we would be able to continuously transmit P numbers per second by means of P' numbers per second, although according to equalities (24) and (25) and inequality (23) we have

$$P > P'\,.$$

Therefore, the assumption that the second part of Theorem VII is invalid leads us to an inadmissible, according to the lemma proven, result.

Transmission capacity in the telephone communication

A conversation, music, and other objects of telephone communication are arbitrary functions of time, which comprise a frequency spectrum whose width is quite definite and depends on how adequately we wish to transmit the sound.

When transmitting this function by wire or by radio, we transform it into another time function which is actually transmitted. In this case, the latter function should necessarily, according to Theorem VII, possess a frequency spectrum of width not smaller than the sound frequency band we would like to transmit.

Therefore, a continuous telephone communication cannot occupy in ether or wire a narrower frequency range than the width of the sound frequency spectrum required for a given broadcast. This is true irrespective of the method of transmission, and it is impossible to contrive a method that would enable occupying a narrower frequency range for continuous transmission.

As is well known, such a minimal frequency spectrum may be afforded even at present by one sideband transmission.

The reservation about 'continuous transmission' is of paramount importance, because it is possible to transmit some sounds, say music, off and on and thereby occupy a narrower frequency range than the width of the sound spectrum we would receive in this case. To do this it would suffice first to record the transmitted music on phonograph records and then to broadcast from them by rotating them, say, two times slower than during the recording. Then, all frequencies will be two times lower than the regular ones and we will manage to occupy a two-fold narrower frequency range during transmission. Such a broadcast may also be reconstructed by means of a phonograph. Clearly, such a broadcast cannot increase the transmission capacity, because the 'ether' or wire will be occupied all the time, while the broadcast will proceed interrupted.

This is not at variance with Theorem VII, either, for its formulation contains reservations: there is no way of transmitting an 'arbitrary function' and of doing this 'continuously', while in the above broadcast we can either transmit off and on an arbitrary function or transmit uninterruptedly a function not quite arbitrary but possessing breaks known beforehand.

From Theorem VII it also follows that the transmission capacity cannot be increased by employing some selections of a nonfrequency nature (excluding directional antennas) or something else of the kind.

Indeed, if this could be done, then by applying these methods it would be possible to transmit from one place to another, say, n telephone broadcasts simultaneously with the frequency spectra of width Δf each, occupying for this purpose a frequency range narrower than $n\Delta f$.

However, in the course of this transmission the field intensities (or currents in the wire) of different broadcasts would be mixed up into some single function of time with the frequency spectra narrower than $n\Delta f$, which will be perceived by receivers. The result would be that we have managed to transfer n time functions with the frequency ranges of width Δf by means of one function with a frequency range narrower than $n\Delta f$, which is strictly prohibited by Theorem VII.

It is clear from the aforesaid that the ether transmission capacity for a telephone may be increased only by resorting to directional antennas or by broadening the operating frequency range through the use of ultrashort (metric) waves.

Transmission of images and television with all half-shadows

In the transmission of images and in television it is required to transfer the degree of blackness of N elements per second, which is equivalent to the transmission of arbitrary numbers with a rate of N numbers per second. If we want to do this by means of a time function, as is always done, according to Theorem VII it has to occupy a frequency range not narrower than $N/2$ periods per second. Therefore, it is immediately evident that in this case, too, the frequency band cannot be reduced more than is allowed by one sideband transmission. True, even its realization can encounter serious technical difficulties due to phase distortions which may occur during such a transmission.

The frequency band cannot be narrowed by means of some 'grouped image scan' (scanning not over individual elements), either, because with this scan, too, one will have to transfer, although by some other means, the degree of blackening of the same N elements per second, i.e., N arbitrary numbers per second, which is impossible to do with a decreased frequency range.

In this case, too, nonfrequency selection techniques (excluding directional antennas) cannot be helpful for the same reasons as in the telephone communication.

Telegraph transmission and image transmission without half-shadows or with their limited number

In telegraph transmission, as well in the transmission of images without half-shadows or with quite definite preas-

Table

I	II	III
0	0	0
1	0	1
1	1	2
0	1	3

signed half-shadows, once again we are dealing with the transmission of a kind of N elements per second, which is equivalent to the transmission of N numbers per second. However, the number of these elements and hence the magnitude of numbers may assume quite definite preassigned values rather than be quite arbitrary. That is why the above-deduced theorems are not directly applicable to these transmissions, for they deal with the transmission of arbitrary numbers which are absolutely unknown beforehand.

True, it is possible to narrow the frequency range required for these transmissions by an arbitrarily large factor and hence increase, at least theoretically, the transmission capacity also by as many times as desired.

To do this we may proceed as follows: we are to transmit, say, with a rate of N elements per second the elements which may assume the values 0 or 1 and in doing this occupy a frequency range of a width of $N/4$ (in lieu of $N/2$ by Theorem VII). For this purpose, we will transmit two such elements by means of one element (or number), for instance, according to the following table, in which column I gives the value of the first element, column II the value of the second, and column III the value of the element intended for their transmission.

In this way we will be able to transmit N two-valued elements per second by means of $N/2$ four-valued elements per second, which can, according to Theorem VII, be transmitted employing a frequency range of a width of $N/4$.

In practice, this replacement of two elements with one may be effected, for instance, by the scheme of Fig. 4, where F_1 and F_2 are two photoelectric cells or two telegraph apparatuses. In this case, F_1 actuates modulator M_1 which sends into the line an amplitude equal to unity, while F_2 operates with modulator M_2 which sends an amplitude equal to 3. In the simultaneous operation of F_1 and F_2, both modulators are actuated and, since they are engaged in opposition, an amplitude equal to 2 is sent. During reception, the signal is fed to three receivers. The first, R_1, is actuated by the amplitude 1, the second, R_2, by the amplitude 2, and the third, R_3, by the amplitude 3. The first receiver R_1 actuates L_1, the second one actuates L_2, and the third one, on arrival of the amplitude equal to 3, denies access to L_1 for the first receiver. By means of this circuit we will obtain the above narrowing of the frequency band.

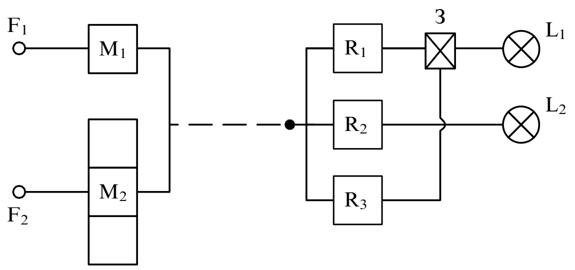

Figure 4.

In the course of such a transmission it is required to distinguish four gradations of the signals under detection instead of two, which evidently generates the need for raising the transmitter power by a factor of $3^2 = 9$ in comparison with conventional transmission.

In a similar way, it is also possible to narrow the frequency band by a factor n by transmitting n elements that may assume two values each with the use of a single element which should evidently be able to assume 2^n values (in accordance with the number of combinations of n elements that may assume two values). Such a transmission calls for a $(2^n - 1)^2$-fold rise in power.

In the transmission of images with a certain number of preassigned half-shadows, every element should be able to assume several, say m (in this case, $m > 2$), values. To narrow the frequency band by a factor n in this transmission, it is possible to replace n transmitted elements with one which should be able to assume m^n values (in accordance with the number of possible combinations of n elements possessing m possible values each). In this case, evidently, the power is to be raised by a factor of $[(m^n - 1)^2]/[(m - 1)^2]$.

One can see that such-like narrowing of the frequency band calls for an enormous increase in power.

Furthermore, the methods described above would fail in transmission at short wavelengths due to fading.

For wire communication, this method of frequency band narrowing may be of practical significance even now, because the powers required in this case are small and there are no fast changes in reception strength.

Appendix I

We substitute expression (7) in Eqn (4) to obtain

$$F(t) = \int_0^{\omega_1} \sum_{-\infty}^{+\infty} D_k \cos \frac{\pi}{\omega_1} k\omega \cos \omega t \, d\omega + \int_0^{\omega_1} \sum_{-\infty}^{+\infty} D_k \sin \frac{\pi}{\omega_1} k\omega \sin \omega t \, d\omega$$

$$= \sum_{-\infty}^{+\infty} D_k \int_0^{\omega_1} \left(\cos \frac{\pi}{\omega_1} k\omega \cos \omega t + \sin \frac{\pi}{\omega_1} k\omega \sin \omega t \right) d\omega = \sum_{-\infty}^{+\infty} D_k \int_0^{\omega_1} \left(\cos \omega \left(t - \frac{\pi}{\omega_1} k \right) \right) d\omega \,,$$

or, upon integrating and replacing ω_1 with $2\pi f_1$ in parentheses, we arrive at

$$F(t) = \sum_{-\infty}^{+\infty} D_k \frac{\sin \omega_1 \big(t - k/(2f_1) \big)}{t - k/(2f_1)} \,.$$

Appendix II

In the expression

$$C(\omega) = \frac{1}{\pi} \int_{k/(2f_1)-T}^{k/(2f_1)+T} D_k \frac{\sin \omega_1 \big(t - k/(2f_1) \big)}{t - k/(2f_1)} \cos \omega t \, dt$$

we make a substitution

$$t = u + \frac{k}{2f_1} \,, \quad dt = du \,,$$

then

$$C(\omega) = \frac{1}{\pi}\int_{-T}^{T} D_k \frac{\sin\omega_1 u}{u}\cos\omega\left(u + \frac{k}{2f_1}\right) du$$
$$= \frac{1}{\pi}\int_{-T}^{T} D_k \frac{\sin\omega_1 u \cos\omega u}{u}\cos\omega\frac{k}{2f_1} du$$
$$+ \frac{1}{\pi}\int_{-T}^{T} D_k \frac{\sin\omega_1 u \sin\omega u}{u}\sin\omega\frac{k}{2f_1} du\,.$$

In passing through the zero, the integrand of the second integral changes its sign, while retaining its magnitude, and therefore the second integral is equal to zero.

With $-u$ in place of u, the integrand of the first integral remain invariable, and therefore this integral may be taken between the limits 0 and T and then multiplied by a factor of two. So, one finds

$$C(\omega) = \frac{2D_k}{\pi}\cos\omega\frac{k}{2f_1}\int_0^T \frac{\sin\omega_1 u\cos\omega u}{u} du\,,$$

or

$$C(\omega) = \frac{D_k}{\pi}\cos\omega\frac{k}{2f_1}\left[\int_0^T \frac{\sin(\omega_1+\omega)u}{u} du - \int_0^T \frac{\sin(\omega-\omega_1)u}{u} du\right].$$

In the first integral we make the following change

$$(\omega_1 + \omega)u = y\,,$$

and in the second one

$$(\omega - \omega_1)u = y\,,$$

to obtain

$$C(\omega) = \frac{D_k}{\pi}\cos\omega\frac{k}{2f_1}\left[\int_0^{(\omega+\omega_1)T} \frac{\sin y}{y} dy - \int_0^{(\omega-\omega_1)T} \frac{\sin y}{y} dy\right].$$

The integrals in square brackets cannot be taken. Clearly, they are some functions of the upper limit. These functions are commonly referred to as integral sines. On introducing this notion we obtain

$$C(\omega) = \frac{D_k}{\pi}\cos\omega\frac{k}{2f_1}\left[\mathrm{Si}\,T(\omega+\omega_1) - \mathrm{Si}\,T(\omega-\omega_1)\right].$$

Doing precisely the same operations on $S(\omega)$, we arrive at Eqn (11).

Appendix III

We substitute equations (14) into Eqn (2) to obtain

$$F(t) = \int_{\omega_1}^{\omega_2}\sum_{-\infty}^{+\infty} D_k \cos\frac{\pi k(\omega-\omega_1)}{\omega_2-\omega_1}\cos\omega t\, d\omega$$
$$+ \int_{\omega_1}^{\omega_2}\sum_{-\infty}^{+\infty} D_k \sin\frac{\pi k(\omega-\omega_1)}{\omega_2-\omega_1}\sin\omega t\, d\omega\,.$$

The limits were taken to be equal to ω_1 and ω_2, because

$$C(\omega) = S(\omega) = 0$$

for

$$\omega < \omega_1 \quad\text{or}\quad \omega > \omega_2\,.$$

Upon trigonometric rearrangement, one finds

$$F(t) = \sum_{-\infty}^{+\infty} D_k \int_{\omega_1}^{\omega_2}\cos\left[\omega\left(t - \frac{\pi k}{\omega_2-\omega_1}\right) + \frac{\pi k\omega_1}{\omega_2-\omega_1}\right] d\omega$$
$$= \sum_{-\infty}^{+\infty} D_k \frac{\sin\left[\omega_2[t - \pi k/(\omega_2-\omega_1)] + \pi k\omega_1/(\omega_2-\omega_1)\right]}{t - \pi k/(\omega_2-\omega_1)}$$
$$- \frac{\sin\left[\omega_1[t - \pi k/(\omega_2-\omega_1)] + \pi k\omega_1/(\omega_2-\omega_1)\right]}{t - \pi k/(\omega_2-\omega_1)}\,.$$

By replacing the difference of sines with a product and performing simplifications, we obtain

$$F(t) = 2\sum_{-\infty}^{+\infty} D_k \cos\left(\frac{\omega_2+\omega_1}{2}t - \frac{\pi}{2}k\right)$$
$$\times \frac{\sin\left[(\omega_2-\omega_1)/2\{t - k/[2(f_2-f_1)]\}\right]}{t - k/[2(f_2-f_1)]}\,,$$

or, grouping together the terms with even and odd k, we arrive at Eqn (15).

Conclusions

(1) In view of the present-day 'tightness in the ether' and in connection with the further rapid progress of radio engineering, especially with the development of short-wavelength telephone communication and image transmission, the task of searching for ways to increase the transmission capacity of 'ether' should be set before research institutes as a burning problem.

The problem of increasing the transmission capacity of wire is also of great economic significance and, therefore, should also be brought under study.

(2) Since there are no ways to increase the transmission capacity of 'ether' or wire during image transmission and telephone communication (for instance, by narrowing the frequency bands of separate broadcasts or by using some methods to separate the broadcasts with overlapping frequencies, etc.) to a greater degree than is allowed by the ordinary transmission with one sideband, all attempts in this area are unrealizable and should be abandoned.

(3) For telegraphy and image transmission without half-shadows or with their limited number, the transmission capacities may theoretically be made as high as desired, but this is associated with a major increase in power and complicated equipment. It is therefore believed that in the near future this frequency band narrowing may find use only in wire communications, where this problem should be explored.

(4) As regards the first category of transmissions (telephone and the transfer of images with half-shadows), all efforts should go into the development of methods of reception and transmission on one sideband as the methods which permit the most efficient use of 'ether' and wire.

The purpose of this development is to improve and simplify the equipment, which is quite complicated at the present time.

(5) An investigation should be made into the problem of increasing the transmission capacity of 'ether' by means of directional antennas, both receiving and transmitting antennas.

(6) The operating frequency range in 'ether' should be broadened by employing, where possible, ultrashort waves and by studying this frequency range.

(7) There is a need to examine the feasibility of improving the frequency stability of radio stations, which will allow a greater compactness in 'ether'.